BT-MRE-I & II-05.16-0748 Reg. No. | | | | | | | | |

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B. Tech. Degree I &II Semester Examination in Marine Engineering May 2016

MRE 1101 ENGINEERING MATHEMATICS I

Time: 3 Hours Maximum Marks: 100

(5 × 20 = 100)

I. (a) Evaluate $\lim_{x\to 0} \frac{x\cos x - \log(1+x)}{x^2}$. (6)

(b) Find c using Mean value theorem where $f(x) = x^2 + 2x + 9$, $a = 1$, $b = 5$ and $a < c < b$. (7)

(c) Find the n[th] derivative of $\frac{1+x^2}{1-x}$. (7)

OR

II. (a) Find the radius of curvature of the curves: $x = a\cos^3\theta$, $y = a\sin^3\theta$. (7)

(b) Find the asymptote of the curve $y^3 = x^2(2a - x)$, (6)

(c) If $y = \tan^{-1} x$, prove that $(1+x^2)y_{n+1} - 2nxy_n + n(n-1)y_{n-1} = 0$. (7)

III. (a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove by Euler's theorem, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (7)

(b) If $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$, then prove that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}$. (7)

(c) Find the relative error in the volume of a right circular cone due to small errors δ_h and δ_r in its height h and radius r respectively. (6)

OR

IV. (a) Discuss the maxima and minima of $x^3 + y^3 - 3axy$. (8)

(b) Prove that of all rectangular parallelopipeds of same volume, the cube has least surface. (7)

(c) Write a note on Brachistochrome problem. (5)

V. (a) Derive the standard equation of hyperbola. (6)

(b) Find the equation of chord joining t_1 and t_2 on the parabola $y^2 = 4ax$. (7)

(c) Find the condition that the line $y = mx + c$ touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (7)

OR

(P.T.O.)

VI. (a) Show that the line $\ell x + my + n = 0$ is a normal to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if (7)

$$\frac{a^2}{\ell^2} + \frac{b^2}{m^2} = \frac{\left(a^2 - b^2\right)^2}{n^2}.$$

(b) Prove that the orthocentre of any triangle inscribed in a rectangular hyperbola on the curve. (7)

(c) Find the asymptotes of hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$. (6)

VII. (a) Find the reduction formula for $\int \sin^m x\,dx$. (10)

(b) Find the volume of solid formed by rotation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about y axis. (10)

OR

VIII. (a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \left(x + y + z + 1\right)^2 dx\,dy\,dz$. (10)

(b) Evaluate $\iint xy\,dx\,dy$ over positive quadrant of circle $x^2 + y^2 = a^2$. (10)

IX. (a) Prove that $f(r)\bar{r}$ is irrotational. (6)

(b) Prove that $\left[\bar{b} \times \bar{c},\ \bar{c} \times \bar{a},\ \bar{a} \times \bar{b}\right] = \left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$ (7)

(c) If φ is a scalar point function, prove that Curl (grad φ) = 0. (7)

OR

X. (a) Prove that $\nabla r^n = n(n+1)r^{n-2}$. (7)

(b) Prove that $\bar{f} = (z\cos x + \sin y)i + (x\cos y + \sin z)j + (y\cos z + \sin x)k$ is irrotational. Also find its scalar potential. (8)

(c) Define linear independence of vectors. (5)

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